researchED Amsterdam 2016

University of Southampton

Wiskunde: vaardigheden, inzicht of allebei?

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researchEd Amsterdam
30 Januari 2016

Inhoud

- Historische achtergrond ('Math Wars')
- Een selectie van wat er gezegd is en wordt over vaardigheden en inzicht (en bv. Algoritmes)
- Vaardigheden en inzicht gaan hand in hand (iteratief)
- Internationaal perspectief: PISA en TIMSS
- Onbenoemde vaardigheid: leesvaardigheid
- Waar gaat het heen?

(U mag inbreken voor vragen)

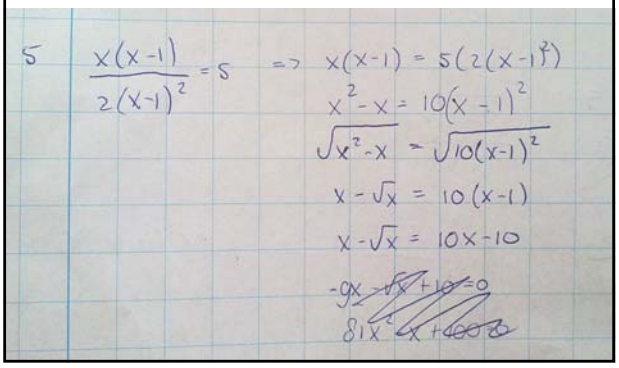
Disclaimer: termen soms naar Nederlands vertaald, maar soms ook Engels gelaten. Opent op zichzelf weer discussie. En: wat is 'inzicht' eigenlijk?

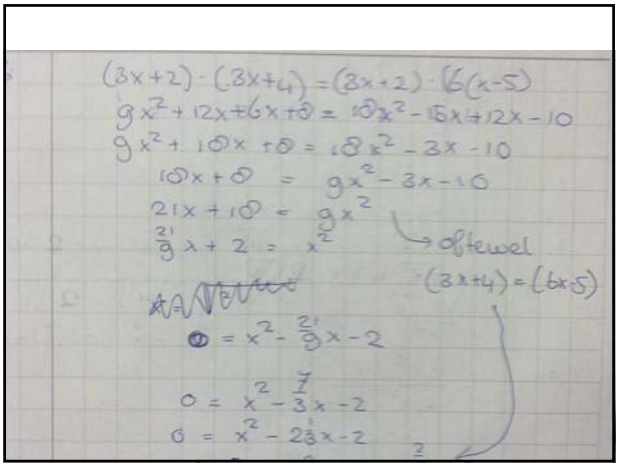
Allebei

Discussie nog steeds actueel

- Christian Bokhove
- 14 jaar lang docent St. Michaël College te Zaandam (havo/vwo) geweest
- Projecten, gepromoveerd in 2011
- Werk nu in Zuid-Engeland, Universiteit van Southampton

In eigen onderzoek





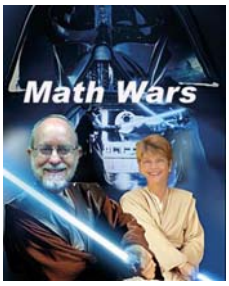
Quantitative Literacy vs. Calculus Preparation

Theorie vs. Toepassingen

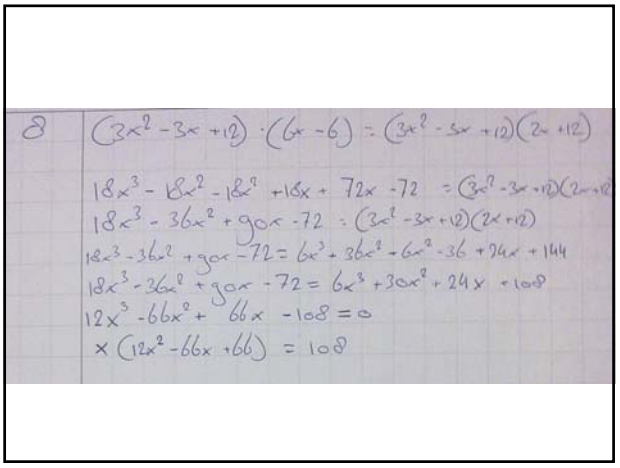
'Rote' vs. 'Constructivisme'

Tracking vs. Mainstreaming

Etc.

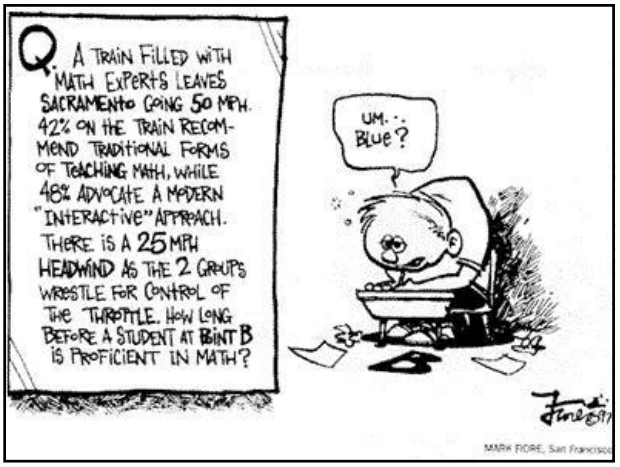


Schoenfeld, A. H. (2004). *The math wars*. Educational Policy, 18 (1), 253–286.

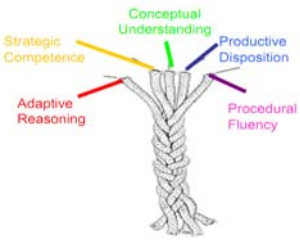


Procedural en Conceptual knowledge

- Historische discussie hoe formeel onderwijs het beste wiskundige expertise kan ondersteunen.
- Vaardigheden versus Inzicht.
- Moeten rekenvaardigheden (alleen) gememoriseerd worden of (alleen) inzichtelijk
- Wereldwijde 'math reform' inspanningen.
- Adaptive vs Routine expertise.



Voorbeeld van een beeld



Conceptual understanding is "the comprehension of mathematical concepts, operations, and relations". Procedural fluency is de "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (p. 116). Furthermore, "the five strands are interwoven and interdependent in the development of proficiency in mathematics" (ibid.). NRC (2001)

Hiebert en Lefevre (1986)

- **Procedural knowledge (knowing how)**
  - Formal mathematical language, algorithms and rules for solving mathematical problems.
  - Generates problem solving behaviour which is not always meaningful and generalizable.
- **Conceptual knowledge (knowing why)**
  - Product of a linking process, which creates relationships between existing knowledge and information that is newly learned.
  - Can be implicit or explicit, however it is flexible, not tied to specific problem contexts and is therefore generalizable.

Nog een voorbeeld: algoritmes

- Fan & Bokhove (2014)
- (Standaard-)algoritmes vaak negatief daglicht: regels, laag niveau, geen inzicht
- Canada: helemaal niet, sommige curricula niet mee expliciet genoemd
- Kamii and Dominick (1997) ‘‘Algorithms are harmful to children’s development of numerical reasoning for two reasons: (a) they ‘unteach’ place value and discourage children from developing number sense, and (b) they force children to give up on their own thinking’’(p. 58).

Algebraic Expertise

Basic skills	**Symbol sense**
• Procedural work	• Strategic work
• Local focus	• Global focus
• Algebraic calculation	• Algebraic reasoning

Arcavi (2005). Symbol sense.

Bokhove & Drijvers (2010). Terugblikkend, straalt te veel 'zwart/wit' uit?

Structure sense, e.g. Hoch & Dreyfus (2004)

Maar…

- Dahlin and Watkins (2000), link memorization and understanding door ‘‘repetition’’.
- Meaningful repetition can ‘‘create a deep impression’’ which leads to memorization, and it can also lead to ‘‘discovering new meaning’’ which in turn leads to understanding (Li, 1999).
- En ook al lang bekend: "many different kinds of procedures and the quality of the connections within a procedure varies" (Anderson 1982)

Bijvoorbeeld voor woordproblemen

- Herkennen wiskundige structuur
- Schemas
- Domain or context specific knowledge structures that organize knowledge and help the learner categorize various problem types to determine the most appropriate actions needed to solve the problem
- E.g. Chen (1999) en Sweller, Chandler, Tierney, & Cooper (1990)

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Star: andere kijk

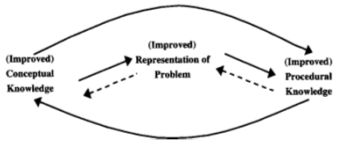
Star (2005) beschreef procedural understanding in termen van 'kwaliteit van kennis' (knowledge quality, De Jong and Ferguson-Hessler 1996).

Table 1
Types and Qualities of Procedural and Conceptual Knowledge

Knowledge type	Knowledge quality	
	Superficial	Deep
Procedural	Common usage of *procedural knowledge*	?
Conceptual	?	Common usage of *conceptual knowledge*

Star, J. R. (2005). Reconceptualizing procedural knowledge. Journal for Research in Mathematics Education, 36(5), 404–411.

Het 'iteratieve' model
(Rittle-Johnson, Siegler, & Alibali, 2001)



Procedurele en conceptuele kennis ontwikkelen gradueel en hand-in-hand

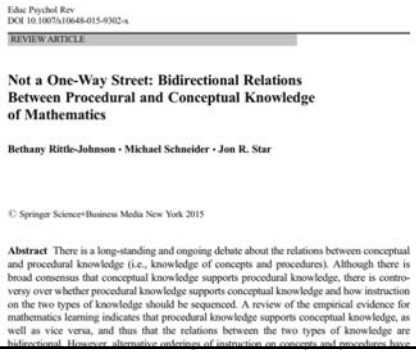
Causale relaties, twee richtingen

Ook kijken naar de ***processen***

Rittle-Johnson, B., Siegler, R.S., and Alibali, M.W. (2001). Developing conceptual understanding and procedural skill in mathematics: an iterative process. *Journal of Educational Psychology, 93*, 2, 346-362.

Source: http://www.bbc.co.uk/news/education-25187997

Rittle-Johnson and Star (2015)



Educ Psychol Rev
DOI 10.1007/s10648-015-9302-x

REVIEW ARTICLE

Not a One-Way Street: Bidirectional Relations Between Procedural and Conceptual Knowledge of Mathematics

Bethany Rittle-Johnson · Michael Schneider · Jon R. Star

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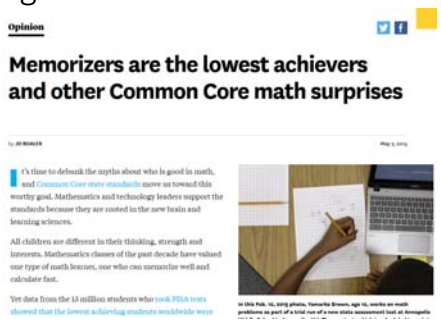
Abstract There is a long-standing and ongoing debate about the relations between conceptual and procedural knowledge (i.e., knowledge of concepts and procedures). Although there is broad consensus that conceptual knowledge supports procedural knowledge, there is controversy over whether procedural knowledge supports conceptual knowledge and how instruction on the two types of knowledge should be sequenced. A review of the empirical evidence for mathematics learning indicates that procedural knowledge supports conceptual knowledge, as well as vice versa, and thus that the relations between the two types of knowledge are bidirectional. However, alternative orderings of instruction on concepts and procedures have

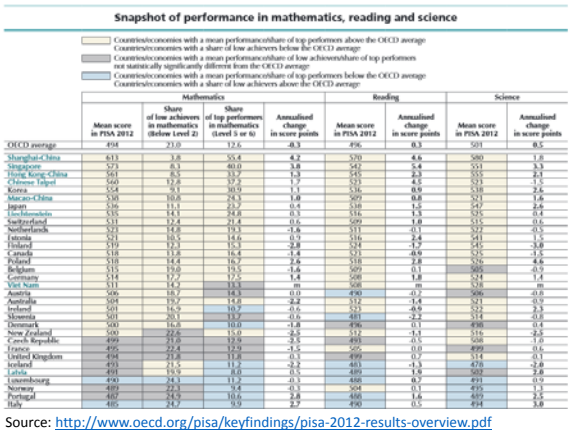
Zegt PISA dit echt?



Source: http://hechingerreport.org/memorizers-are-the-lowest-achievers-and-other-common-core-math-surprises/

Wat zeggen internationale vergelijkingen?

- Veel beperkingen!
- Data verzameld zonder onderzoeksvraag
- Secundaire data analyse
- Maar: enige manier grootschalige, internationale vergelijkingen.
- Meer weten over beperkingen? Artikel hierover ingeleverd (focus PISA)



Snapshot of performance in mathematics, reading and science

Countries/economies with a mean performance/share of top performers above the OECD average
Countries/economies with a share of low achievers below the OECD average
Countries/economies with a mean performance/share of low achievers/share of top performers not statistically significantly different from the OECD average
Countries/economies with a mean performance/share of top performers below the OECD average
Countries/economies with a share of low achievers above the OECD average

	Mathematics				Reading		Science	
	Mean score in PISA 2012	Share of low achievers in mathematics (Below Level 2)	Share of top performers in mathematics (Level 5 or 6)	Annualised change in score points	Mean score in PISA 2012	Annualised change in score points	Mean score in PISA 2012	Annualised change in score points
OECD average	494	23.0	12.6	-0.3	496	0.3	501	0.5
Shanghai-China	613	3.8	55.4	4.2	570	6.6	580	1.0
Singapore	573	8.3	40.0	3.8	542	5.4	551	3.3
Hong Kong-China	561	8.5	33.7	1.3	545	2.3	555	2.1
Chinese Taipei	560	12.8	37.3	1.7	523	4.5	523	-1.5
Korea	554	9.1	30.9	1.1	536	0.9	538	2.6
Macao-China	538	10.8	24.3	1.8	509	0.8	521	1.6
Japan	536	11.1	23.7	0.4	538	1.5	547	2.6
Liechtenstein	535	14.1	24.8	0.3	516	1.3	535	0.4
Switzerland	531	12.4	21.4	0.6	509	1.0	515	0.6
Netherlands	523	14.8	19.3	-1.6	511	-0.1	522	-0.5
Estonia	521	10.5	11.6	0.9	516	2.4	541	1.5
Finland	519	12.1	15.3	-2.8	524	-1.7	545	-3.0
Canada	518	13.8	16.4	-1.4	523	-0.9	525	-1.5
Poland	518	14.4	16.7	2.6	518	2.8	526	4.6
Belgium	515	19.0	19.5	-1.6	509	0.1	505	-0.9
Germany	514	17.7	17.5	1.4	508	1.8	524	1.4
Viet Nam	511	14.2	13.3	m	508	m	528	m
Austria	506	18.7	14.3	0.0	490	-0.2	506	-0.8
Australia	504	19.7	14.8	-2.2	512	-1.4	521	-0.9
Ireland	501	16.9	10.7	-0.6	523	-0.9	522	2.3
Slovenia	501	20.1	13.2	-0.6	481	-2.2	514	-0.8
Denmark	500	16.8	10.0	-1.8	496	0.1	498	0.4
New Zealand	500	22.6	15.0	-2.5	512	-1.1	516	-2.5
Czech Republic	499	21.0	12.9	-2.5	493	-0.5	508	1.0
France	495	22.4	12.9	-1.5	505	0.0	499	0.6
United Kingdom	494	21.8	11.8	-0.1	499	0.7	514	-0.1
Iceland	493	21.5	11.2	-2.2	483	-1.3	478	-2.0
Latvia	491	19.9	8.0	0.5	489	1.9	502	2.0
Luxembourg	490	24.3	11.2	-0.3	488	0.7	491	0.9
Norway	489	22.1	9.4	-0.1	504	0.1	495	1.3
Portugal	487	24.9	10.6	2.8	488	1.6	489	2.5
Italy	485	24.7	9.9	2.7	490	0.5	494	3.0

Source: http://www.oecd.org/pisa/keyfindings/pisa-2012-results-overview.pdf

Question memorization?

Q41 **For each group of three items, please choose the item that best describes your approach to mathematics.**

a) Please tick only one of the following three boxes.

1. When I study for a mathematics test, I try to work out what the most important parts to learn are.
2. When I study for a mathematics test, I try to understand new concepts by relating them to things I already know.
3. When I study for a mathematics test, I learn as much as I can off by heart.

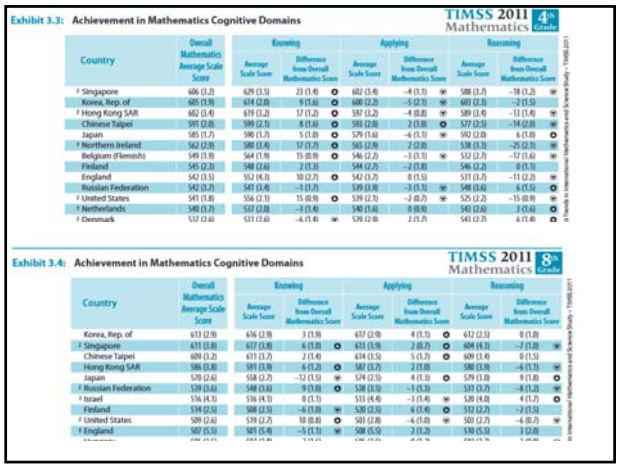


Exhibit 3.3: Achievement in Mathematics Cognitive Domains

TIMSS 2011 Mathematics 4th Grade

Country	Overall Mathematics Average Scale Score	Knowing		Applying		Reasoning	
		Average Scale Score	Difference from Overall Mathematics Score	Average Scale Score	Difference from Overall Mathematics Score	Average Scale Score	Difference from Overall Mathematics Score
‡ Singapore	606 (3.2)	629 (3.5)	23 (1.4) ○	602 (3.4)	−4 (1.3) ▼	588 (3.7)	−18 (1.2) ▼
Korea, Rep. of	605 (1.9)	614 (2.0)	9 (1.6) ○	600 (2.2)	−5 (2.1) ▼	601 (2.3)	−2 (1.5)
‡ Hong Kong SAR	602 (3.4)	619 (3.2)	17 (1.2) ○	597 (3.2)	−4 (0.8) ▼	589 (3.4)	−13 (1.4) ▼
Chinese Taipei	591 (2.8)	599 (2.3)	8 (1.6) ○	593 (2.8)	2 (1.8) ○	577 (2.5)	−14 (2.8) ▼
Japan	585 (1.7)	590 (1.7)	5 (1.0) ○	579 (1.6)	−6 (1.1) ▼	592 (2.0)	6 (1.0) ○
† Northern Ireland	562 (2.9)	580 (3.4)	17 (1.7) ○	565 (2.9)	2 (2.0)	538 (3.3)	−25 (2.1) ▼
Belgium (Flemish)	549 (1.9)	564 (1.9)	15 (0.9) ○	546 (2.2)	−3 (1.3) ▼	532 (2.7)	−17 (1.6) ▼
Finland	545 (2.3)	548 (2.6)	2 (1.3)	544 (2.7)	−2 (1.8)	546 (3.2)	0 (1.3)
England	542 (3.5)	552 (4.3)	10 (2.7) ○	542 (3.7)	0 (1.5)	531 (3.7)	−11 (2.2) ▼
Russian Federation	542 (3.7)	541 (3.4)	−1 (1.7)	539 (3.9)	−3 (1.1) ▼	548 (3.6)	6 (1.5) ○
‡ United States	541 (1.8)	556 (2.1)	15 (0.9) ○	539 (2.1)	−2 (0.7) ▼	525 (2.2)	−15 (0.9) ▼
† Netherlands	540 (1.7)	537 (2.8)	−3 (1.4)	540 (1.6)	0 (0.9)	541 (2.6)	1 (1.6) ○
‡ Denmark	537 (2.6)	533 (2.6)	−4 (1.4) ▼	539 (2.9)	2 (1.2)	541 (2.7)	4 (1.4) ○

Exhibit 3.4: Achievement in Mathematics Cognitive Domains

TIMSS 2011 Mathematics 8th Grade

Country	Overall Mathematics Average Scale Score	Knowing		Applying		Reasoning	
		Average Scale Score	Difference from Overall Mathematics Score	Average Scale Score	Difference from Overall Mathematics Score	Average Scale Score	Difference from Overall Mathematics Score
Korea, Rep. of	613 (2.9)	616 (2.9)	3 (1.9)	617 (2.9)	4 (1.3) ○	612 (2.5)	0 (1.0)
‡ Singapore	611 (3.8)	617 (3.8)	6 (1.0) ○	613 (3.9)	2 (0.7) ○	604 (4.3)	−7 (1.0) ▼
Chinese Taipei	609 (3.2)	611 (3.7)	2 (1.4)	614 (3.5)	5 (1.7) ○	609 (3.4)	0 (1.5)
Hong Kong SAR	586 (3.8)	591 (3.9)	6 (1.7) ○	587 (3.7)	2 (1.0)	580 (3.9)	−6 (1.1) ▼
Japan	570 (2.6)	558 (2.7)	−12 (1.5) ▼	574 (2.5)	4 (1.3) ○	579 (3.0)	9 (1.8) ○
‡ Russian Federation	539 (3.6)	548 (3.6)	9 (1.8) ○	538 (3.5)	−1 (1.3)	531 (3.7)	−8 (1.2) ▼
† Israel	516 (4.1)	516 (4.1)	0 (1.1)	511 (4.4)	−3 (1.4) ▼	520 (4.0)	4 (1.7) ○
Finland	514 (2.5)	508 (2.5)	−6 (1.0) ▼	520 (2.5)	6 (1.4) ○	512 (2.7)	−2 (1.5)
‡ United States	509 (2.6)	519 (2.7)	10 (0.8) ○	503 (2.8)	−6 (1.0) ▼	503 (2.7)	−6 (0.7) ▼
† England	507 (5.5)	501 (5.4)	−5 (1.1) ▼	508 (5.5)	2 (1.2)	510 (5.5)	3 (2.0)

TIMSS

The good scores of East Asian students seem to contradict the assumption that Asian classrooms are traditional and aimed mainly at low-level cognitive goals (e.g. emphasizing memorization), which has been called a ‘‘Paradox of the Asian learner’’ (Biggs 1994, 1998).

Rol van taal

- Aantal woorden
- Vocabulaire
- Symbolische taal
- Visualisaties

Hell's library

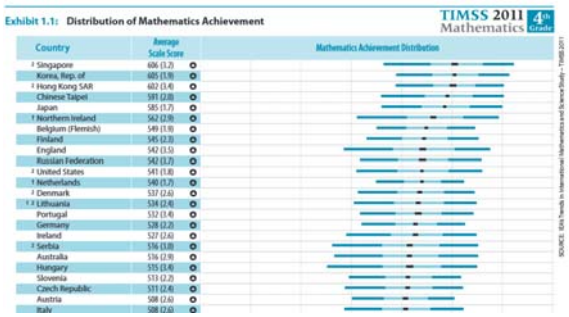


Exhibit 1.1: Distribution of Mathematics Achievement

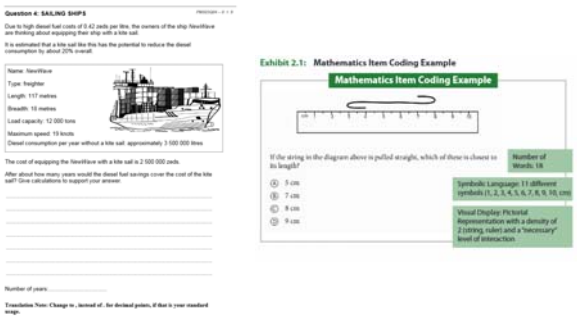
TIMSS 2011 Mathematics 4th Grade

Country	Average Scale Score	Mathematics Achievement Distribution
‡ Singapore	606 (3.2)	○
Korea, Rep. of	605 (1.9)	○
‡ Hong Kong SAR	602 (3.4)	○
Chinese Taipei	591 (2.8)	○
Japan	585 (1.7)	○
† Northern Ireland	562 (2.9)	○
Belgium (Flemish)	549 (1.9)	○
Finland	545 (2.3)	○
England	542 (3.5)	○
Russian Federation	542 (1.7)	○
‡ United States	541 (1.8)	○
† Netherlands	540 (1.7)	○
‡ Denmark	537 (2.6)	○
‡ † Lithuania	534 (2.4)	○
Portugal	532 (1.4)	○
Germany	528 (2.2)	○
Ireland	527 (2.6)	○
‡ Serbia	516 (3.8)	○
Australia	516 (2.9)	○
Hungary	515 (3.4)	○
Slovenia	513 (2.2)	○
Czech Republic	511 (2.4)	○
Austria	508 (2.6)	○
Italy	508 (2.6)	○

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS 2011

Source: http://timssandpirls.bc.edu/timss2011/downloads/T11_IR_M_Chapter1.pdf

Voorbeeld vragen



Question 4: SAILING SHIPS

Due to high diesel fuel costs of 0.42 zeds per litre, the owners of the ship NewWave are thinking about equipping their ship with a kite sail.

It is estimated that a kite sail like this has the potential to reduce the diesel consumption by about 20% overall.

Name: NewWave
Type: freighter
Length: 117 metres
Breadth: 18 metres
Load capacity: 12 000 tons
Maximum speed: 19 knots
Diesel consumption per year without a kite sail: approximately 3 500 000 litres

The cost of equipping the NewWave with a kite sail is 2 500 000 zeds.

After about how many years would the diesel fuel savings cover the cost of the kite sail? Give calculations to support your answer.

Number of years: .....

Translation Note: Change to , instead of . for decimal points, if that is your standard usage.

Exhibit 2.1: Mathematics Item Coding Example

Mathematics Item Coding Example

If the string in the diagram above is pulled straight, which of these is closest to its length?

Ⓐ 5 cm
Ⓑ 7 cm
Ⓒ 8 cm
Ⓓ 9 cm

Number of Words: 18

Symbolic Language: 11 different symbols (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, cm)

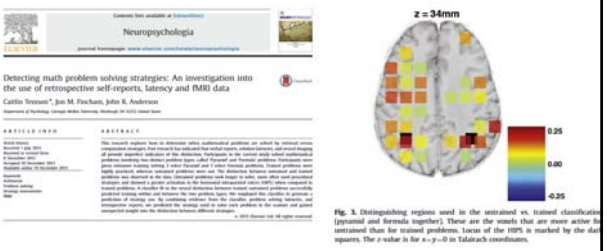
Visual Display: Pictorial Representation with a density of 2 (string, ruler) and a "necessary" level of interaction

http://www.oecd.org/pisa/pisaproducts/pisa2012-2006-rel-items-maths-ENG.pdf

Verschil vragen

- PISA meer problem solving
- Allebei zeer bepaald door IQ, maar PISA meer dan TIMSS (Rindermann, 2007).
- TIMSS meer curriculum georienteerd. (Rindermann & Baumeister, 2015)
- TIMSS-PIRLS relationship report: slechtere lezers deden het slechter dan betere lezers maar extra benadeeld op vragen met meer leeswerk.

Voordelen/nadelen?



Tenison, C., Fincham, J. M., & Anderson, J. R. (2014). Detecting math problem solving strategies: An investigation into the use of retrospective self-reports, latency and fMRI data. Neuropsychologia, 54, 41-52.

Voorbeeld hoe Azie wellicht beide doet: conceptuele variatie

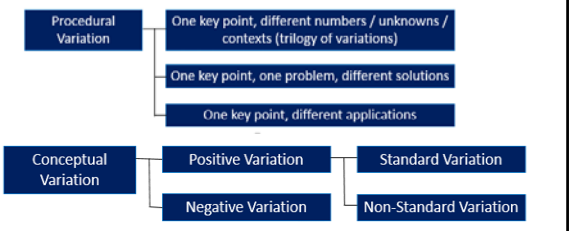


Diagram: Clare Hill from Twynham school

Misschien schakel tussen vaardigheden en inzicht?

Vragen/discussie

Wiskunde: vaardigheden, inzicht of allebei?

Allebei

- Vragen/kritiek/opmerkingen/discussie
- Bedankt
- Twitter: @cbokhove
- Presentatie op www.bokhove.net (inclusief refs)

Waar gaan we heen?

- Focus op zowel vaardigheden en inzicht
- Onderzoek fMRI en algebra, bijvoorbeeld Cognitive Tutor
- Zelf bezig met leesvaardigheid en toetsitems
- Verbanden tussen 'memorisation and understanding' (In Engeland is er Azie hype)
- Sterke verbanden maken tussen onderwijskundig, (neuro-)psychologisch onderzoek en computer science (bv Rotation skills)

References (1)

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